REVIEW

in regards with application process
for the academic position of PROFESSOR in:
The field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics,
Specialty "Numerical Analysis",
Announced in Official Gazette No. 50 of 15.06.2021 for the needs of
The Technical University of Gabrovo,
Department of Mathematics, Informatics and Natural Sciences

The only applicant: Assoc. Prof. PhD Todor Dimitrov Todorov

Reviewer: Prof. Doctor of Sciences Andrey Borisov Andreev

1. Brief details of the applicant

Todor Dimitrov Todorov graduated in Mathematics at the University of Plovdiv "Paisii Hilendarski" in 1986. In the same year he started his career in mathematics in a school in Dryanovo. In 1989, after a competition, he was appointed assistant professor in the Department of Mathematics at the Technical University of Gabrovo, where he has been working ever since. At present he is the head of the Department of Mathematics, Informatics and Natural Sciences (MIMS) at the Technical University of Gabrovo. He was a part-time PhD student at the same university.

In 2001 Todor Todorov successfully defended his dissertation thesis for the degree of PhD on "Isoparametrics in the finite element method". In 2006 he was awarded the academic position of Associate Professor in 01.01.09. "Numerical Analysis" for the needs of Technical University of Gabrovo.

Most of the scientific achievements of Assoc. Prof. Todorov are in the field of numerical methods for partial differential equations and in particular in the finite element method (FEM). His research interests in mathematics are varied. In the field of Numerical Analysis and Scientific Computing, the subject of the present application, the scientific environment is very favourable for him, given the specialities at the university where he teaches.

Associate Professor Todorov is a supervisor of a successful PhD student, and has also for many years supervised a student mathematics team which has had success at national level.

2. Monography presented

The candidate entered the application process with a monograph entitled "Conforming methods for discretization of a bounded domain in multidimensional Euclidean spaces". The monography is formed of 7 scientific publications, refereed and indexed in Scopus and Web of Science. One of them is a single author paper and the remaining 6 papers have one co-author. Five of the paper have an impact factor, and it makes a good impression that most of them have been published in the last 3 years.

3. Contributions to the monographic work

Continuous couplings between elements in a discretization are often called conforming discretization in FEM. The successful solving of this process is a laborious and important procedure, especially when the dimension n of the model problem is large. On the other hand, it is sometimes impossible and even unnecessary to satisfy such a requirement (e.g. in the case of local mesh refinement).

A good impression in the submitted works makes the fact that the candidate does not avoid difficult problems and problems are considered under $n \geq 3$. In these, the classical approach is to connect hypercubic and simplicial meshes in a conforming way. In [A60] a decisive step is taken in this respect by stating the explicit transition elements used.

Simplicial and pyramidal finite elements have undisputed approximation properties in domains with complex geometry and in boundary layer mesh construction. An important result is contained in [A55] for different classes of simplicial elements, and invariance with respect to different refinement approaches has been established.

The papers in this monograph are clearly motivated by important questions of computational mechanics. Therefore, it is a natural aspiration of the research engineer to work with a discretization from a uniform (quasi-uniform) grid. In [A35], for $n=3$, a refinement method was derived that does not "spoil" orthogonal or regular tetrahedra (see also [A50]).

For arbitrary dimension n, an iterative algorithm for intersecting of convex polytopes is given. This result also holds when the elements are of different dimension in n-dimensional space $(1 \leq k < l \leq n)$.

The recapitulation of my acquaintance with the monograph leads me to conclude that the work not only satisfies the scientometric requirements of the competition, but also makes a certain contribution to the numerical solution of important model problems in engineering mechanics and engineering physics.

4. Submitted scientific papers for the application

In addition to the works in the monograph, the candidate Assoc. Todorov has presented 27 more scientific articles. 8 of them are single-authored. The presented publications can be generally referred to the qualitative theory of FEM, results on the numerical implementation of FEM and its applications. I believe that this fully corresponds to the scientific specialty of the announced competition, namely "Numerical Analysis".

5. Contributions of scientific publications

The publications submitted by the candidate can be divided into several thematic areas. Although these strands are not completely independent, I will focus on the main results in each of them.

A) Finite element grids analysis and their implications for the resultant system

The use of hybrid grids is a relatively new approach in FEM. Combining different types of elements helps to conforming coupling of the approximate solution and is a natural combination with the isoparametric approach. The latter means that the transformation from a fixed (basic) element to an arbitrary element of the domain is performed with polynomials of degree equal to the degree of the polynomials with which the approximation is performed. This is dictated by the type of the boundary, which may even contain turning points (lines).

The main results are from the effect of refinement, which has two main aspects:

- Minimizing the degeneracy measure under sequential refinement of simplicial (pyramidal) finite elements [A36, A51]. A new strategy (algorithm) in this direction is proposed in [A35]. In [A57], two theorems for determining the rate of divergence under refinement are proved;
- The solution quality of the resultant algebraic system is another aspect. It depends on the constant γ in the strengthed Cauchy-Bunyakovsky-Schwarz inequality and the cosine of the abstract angle between two subsequent finite element spaces [A53].

B) Non-local boundary value problems

These are most often elliptic problems encountered in physics. Nonlocality is manifested by:

- Presence of a nonlocal multiplier in the model problem;
- Non-local (boundary) conditions between two or more domains.

Associate Professor Todorov considers interesting situations of the first type, whose problems are weakly nonlinear. A variety of analysis tools is used. In [A40] and [A48] sufficient conditions for the existence of a weak solution are given. In [A39] and [A42] FEM is used, and in the first paper it is applied to a nonlocal nonlinear problem for a general second-order elliptic operator, and in the second to a nonlinear parabolic problem. In [A46], an equation with *p*- Laplacian and Dirichlet boundary conditions is considered.

C) FEM for linear boundary value problems

The isoparametric approach provides rich opportunities for better approximation in areas with complex geometry. This method has been applied in [A29] to compute the flow through a curvilinear boundary, and in [A31] to high degree triangular finite elements. The multigrid method is also analyzed ([A56] and [A31]). Numerous numerical experiments are also performed.

For hybrid networks and high-dimensional problems, numerical quadratures are required. They are particularly useful when positive coefficients are used to preserve coercivity in problems with an elliptic operator. Such a formula is presented in [A62]. In [A59], quadratures when $n=4$ are proposed and applied to various transition finite elements. A *21*-point quadrature for a four-dimensional finite element has also been developed [A58].

D) Application of neural networks for DE solving and voice signal classification

This direction is provoked by the topic of the Ph. Todorov's PhD. The solution of a second-order nonlinear elliptic PDE is demonstrated using a basic neural network [A58]. New finite element approaches for voice recognition and signal processing are presented ([A37] and [A38]). In this thematic area, results have been obtained for voice control based on a stochastic classifier (see [A41], [A43], [A44]).

6. Tutorial and pedagogical activities

Associate Professor Todorov has an impressive teaching experience at the Technical University of Gabrovo. For more than 30 years, he has given lectures and exercises in all parts of the "Higher Mathematics" courses for students of engineering, economics and management. He is currently the head of the Department of MIS at the Technical University of Gabrovo.

To the materials for the competition are submitted 3 textbooks with single author Todor Todorov. They cover the material of mathematics, which is studied at TU of Gabrovo. Each title has a large number of problems with their answers. I have been a reviewer of these 3 textbooks and after discussions and comments I can responsibly state that all three books are extremely useful for students.

Assoc. Todorov is also a long-time leader of the representative mathematics team of the Technical University of Gabrovo. He has achieved remarkable success with these teams over the years and has been the reason why several young engineers have chosen a profession actively related to mathematics.

The book "Competitive Mathematics" is also presented to the competition. It contains a selection of original problems from poor mathematics, as well as problems given at national and international mathematics competitions for students.

One of the competitors actively participating in the team - Georgi Tsanev - successfully defended his dissertation under the supervision of Assoc. Todorov on "Research of voice control methods".

7. Citations from other authors

The papers in which Todor Todorov is author or co-author have been cited 68 times. The predominant number of citations are in specialized international journals and/or in journals with impact factor.

8. Critical comments and recommendations

I have no substantive criticisms of the research and teaching presented. I will only make two non-substantive clarifications:

a. Success in conforming connecting of different type finite elements still does not make the method conforming. For example, isoparametrics and numerical integration "spoil" conformality even though they are not called nonconforming methods;
b. I think it should be said "quadratures with positive coefficients" instead of "quadratures with positive weights".

Finally, I would like to make a recommendation in the future work to Assoc. Prof. Todorov not to get distracted in topics that are too far from his main scientific interests.

9. Personal impressions of the candidate

I have known Todor Todorov since his appointment as assistant professor in the Department of Mathematics of the Technical University of Gabrovo in 1989. He has impressed me with his perseverance, persistence and drive to prove himself in his direct work. All of this was underpinned with a certain love for mathematical science and the patience to explain to younger learners. In addition, Todor Todorov enjoys authority; he also has personal contacts with leading foreign scientists.

10. Conclusion

I declare with conviction that the materials submitted for the competition fully comply with the requirements of the Law on the Acquisition of Scientific Degrees of Republic of Bulgaria and Academic Positions procedure requirements at the Technical University - Gabrovo.

Therefore, and in view of the above, I propose that Assoc. Prof. Todor Dimitrov Todorov **be elected** as "Professor" in the Department of MISP, Faculty of Economics, Technical University of Gabrovo in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics, specialty "Numerical Analysis".

29.10.2021 Reviewer: /signature/
Gabrovo /Prof. DSc A. B. Andreev/